

Questão 1:

a) $f(x,y) = x^3y^2 - 2x^2y + e^{xy}$

• $f_x(x,y) = 3x^2y^2 - 4xy + e^{xy} \cdot y$

• $f_y(x,y) = 2x^3y - 2x^2 + x e^{xy}$

b) $f(x,y) = \ln(x^2+y^2) + \sin(xy)$

• $f_x(x,y) = \frac{1}{x^2+y^2} \cdot 2x + \cos(xy) \cdot y$

$$= \frac{2x}{x^2+y^2} + y \cos(xy)$$

• $f_y(x,y) = \frac{2y}{x^2+y^2} + x \cos(xy)$

c) $f(x,y) = \frac{x^2+y}{x-y}$

• $f_x(x,y) = \frac{(2x(x-y) - (x^2+y) \cdot 1)}{(x-y)^2}$
 $= \frac{x^2 - 2xy - y}{(x-y)^2}$

• $f_y(x,y) = \frac{(1 \cdot (x-y) - (x^2+y)(-1))}{(x-y)^2}$
 $= \frac{x + x^2}{(x-y)^2}$

$$d) f(x, y) = e^{x^2 - y^2} + \sqrt{x + y}$$

$$\bullet f_x(x, y) = e^{x^2 - y^2} \cdot (2x) + \frac{1}{2\sqrt{x + y}} \cdot 1$$

$$= 2xe^{x^2 - y^2} + \frac{1}{2\sqrt{x + y}}$$

$$\bullet f_y(x, y) = e^{x^2 - y^2} \cdot (-2y) + \frac{1}{2\sqrt{x + y}} \cdot 1$$

$$= -2ye^{x^2 - y^2} + \frac{1}{2\sqrt{x + y}}$$

Questão 2:

$$x^2y + e^{xz} + y^2z = 4$$

$\frac{\partial z}{\partial x}$:

$$2xy + e^{xz} \cdot (1 \cdot z + x \cdot \frac{\partial z}{\partial x}) + y^2 \frac{\partial z}{\partial x} = 0 \Rightarrow$$

$$2xy + ze^{xz} + xe^{xz} \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial x} = 0 \Rightarrow$$

$$(xe^{xz} + y^2) \frac{\partial z}{\partial x} = - (2xy + ze^{xz}) \Rightarrow$$

$$\frac{\partial z}{\partial x} = - \frac{(2xy + ze^{xz})}{xe^{xz} + y^2}$$

$$\bullet \frac{\partial z}{\partial y} :$$

$$x^2 + e^{xz} \cdot x \frac{\partial z}{\partial y} + 2yz + y^2 \frac{\partial z}{\partial y} = 0 \Rightarrow$$

$$(xe^{xz} + y^2) \frac{\partial z}{\partial y} = -(x^2 + 2yz) \Rightarrow$$

$$\frac{\partial z}{\partial y} = - \frac{(x^2 + 2yz)}{(xe^{xz} + y^2)}$$